Piezoceramic disks with a front layer - Comparisons of FEM with experimental data

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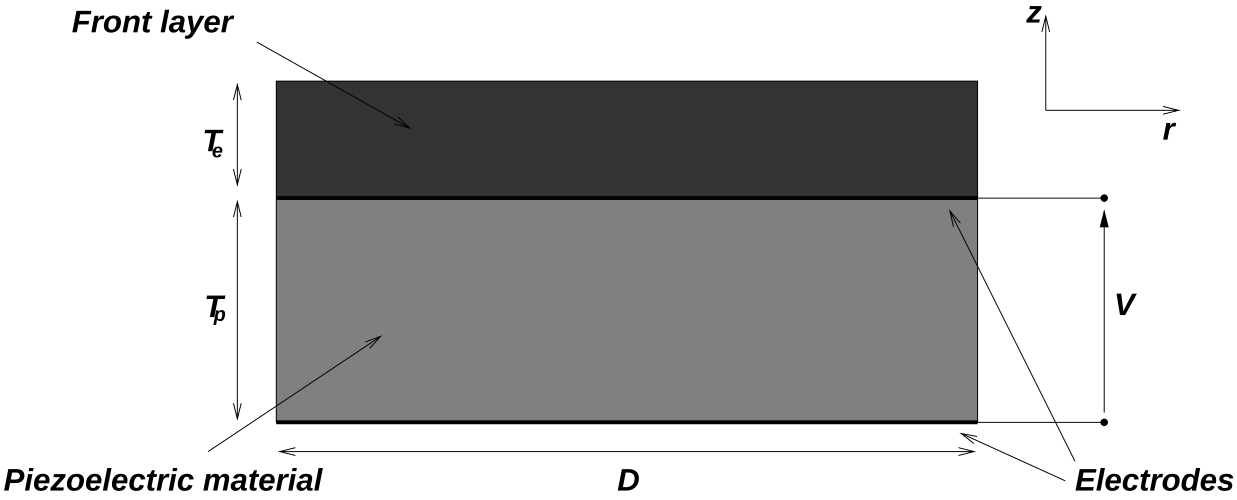
- Motivation
- Introduction
- Theory
- Analysis
  - Piezoceramic disk with a very thin front layer:
    Simulation of mode-shape, conductance and average displacement
  - Piezoceramic disk with thicker front layer:
    Comparison between simulation and measurements
  - Different materials in front layer
- Conclusions

Motivation

- FEM of complete axisymmetric transducer-structure (fluid)
- Self-developed program (FEMP) gives greater flexibility
- Work schedule:
  - Piezoceramic disk, vacuum (Ust., 1997)
  - *Piezoceramic disk with front layer, vacuum (Ust.,1998)*
  - Piezoceramic disk with front/backing layers, vacuum
  - Piezoceramic axisymmetric transducer, vacuum
  - Piezoceramic axisymmetric transducer, fluid (water) (Ph.D. Thesis, 2000)
- Verification against FEM and theory
- Simulations of:
  - Impedance/Admittance
  - Displacement field
  - Stress field (internal)
  - Radiated sound pressure/particle velocity
  - Directivity pattern
  - Sensitivity response

Introduction



- Piezoceramic disk with front layer, varying thickness T_e.
- Front layer: titanium, divinycell and plexiglass
- Simulations:
  - FEM-simulations, self-developed program
  - Uses 8-node isoparametric, axisymmetric elements
  - Loss included using complex constants for piezoceramic and elastic material.
  - Eigenvalues checked with commercial FEM program Abaqus
- Measurements:
  - Erlend Bjørndal, M.Sc. Thesis, 1994
  - Including acoustical and electrical measurements

Theory

FEM for Piezoelectric disk

- Variational principle
- Divide disc into nodal points and elements
- Displacement and potential approximated by nodal values in each local element
- Formulation of the Finite Element Equations

$$\begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix} \begin{bmatrix} \hat{u} \\ \hat{\phi} \end{bmatrix} + \begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\hat{u}} \\ \ddot{\hat{\phi}} \end{bmatrix} = \begin{bmatrix} F \\ -Q \end{bmatrix}$$

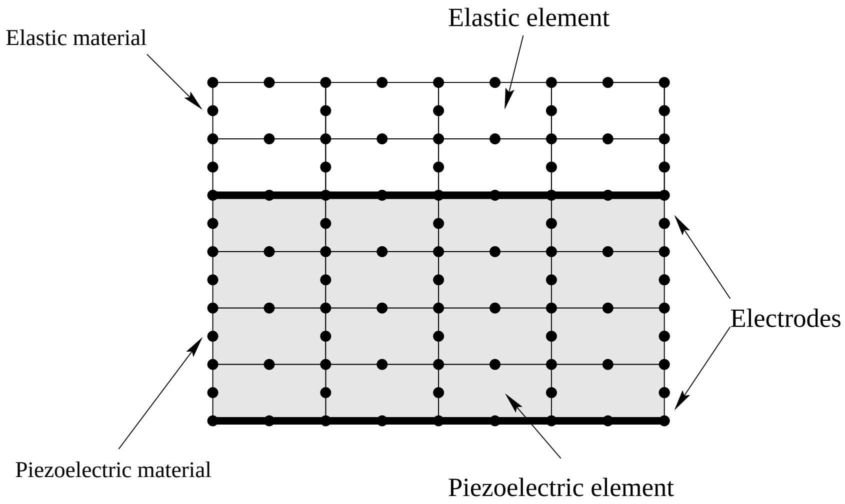
  where for example the stiffness matrix $[K_{uu}]$ is given by:

$$\begin{aligned} [K_{uu}] &= \sum_m [K_{uu}^{(m)}] \\ [K_{uu}^{(m)}] &= \int_{V^{(m)}} [B_u^{(m)}]^T [c^{E(m)}][B_u^{(m)}] dV \end{aligned}$$

- Impedance/Admittance and response calculation:
  - mode-superposition
  - damping implemented using complex constants in eigen-value solve

Theory

Piezoelectric disk with front layer



- Piezoelectric elements:
  - Calculated as for piezoelectric case
- Elastic elements:
  - piezoelectric coupling matrix calculated to zero:

$$\begin{bmatrix} K_{uu} & 0 \\ 0 & K_{\phi\phi} \end{bmatrix} \begin{bmatrix} \hat{u} \\ \hat{\phi} \end{bmatrix} + \begin{bmatrix} M_{uu} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\hat{u}} \\ \ddot{\hat{\phi}} \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

  - Decouples electrical and mechanical variables
  - Equations for electrical potential removed
- Local matrices assembled to global FEM-equations
- System solved as for piezoelectric case

Analysis

- Material parameters for PZT5A:

Constant	Value
c_{11}^E	$12.1 * 10^{10} N/m^2$
c_{12}^E	$7.54 * 10^{10} N/m^2$
c_{13}^E	$7.52 * 10^{10} N/m^2$
c_{33}^E	$11.1 * 10^{10} N/m^2$
c_{55}^E	$2.11 * 10^{10} N/m^2$
e_{31}	$-5.4 C/m^2$
e_{33}	$15.8 C/m^2$
e_{15}	$12.3 C/m^2$
$\epsilon_{11}^S/\epsilon_0$	916
$\epsilon_{33}^S/\epsilon_0$	830
ρ	$7750 kg/m^3$
Q_m	75
$tan\delta_e$	0.02

- Material parameters for frontlayer:

Material	ρ $[kg/m^3]$	E [MPa]	σ	Q_m
Titan	4486	$116.1 * 10^3$	0.32	10000
Plexiglass GS	1190	3300	0.35	80
Divinycell HCP-90	371	980	0.35	80

- Dimensions:

	Diameter	Thickness
PZT5A	12.87 mm	1.005mm / 1.020mm
Frontlayer	12.87 mm	0-12 mm

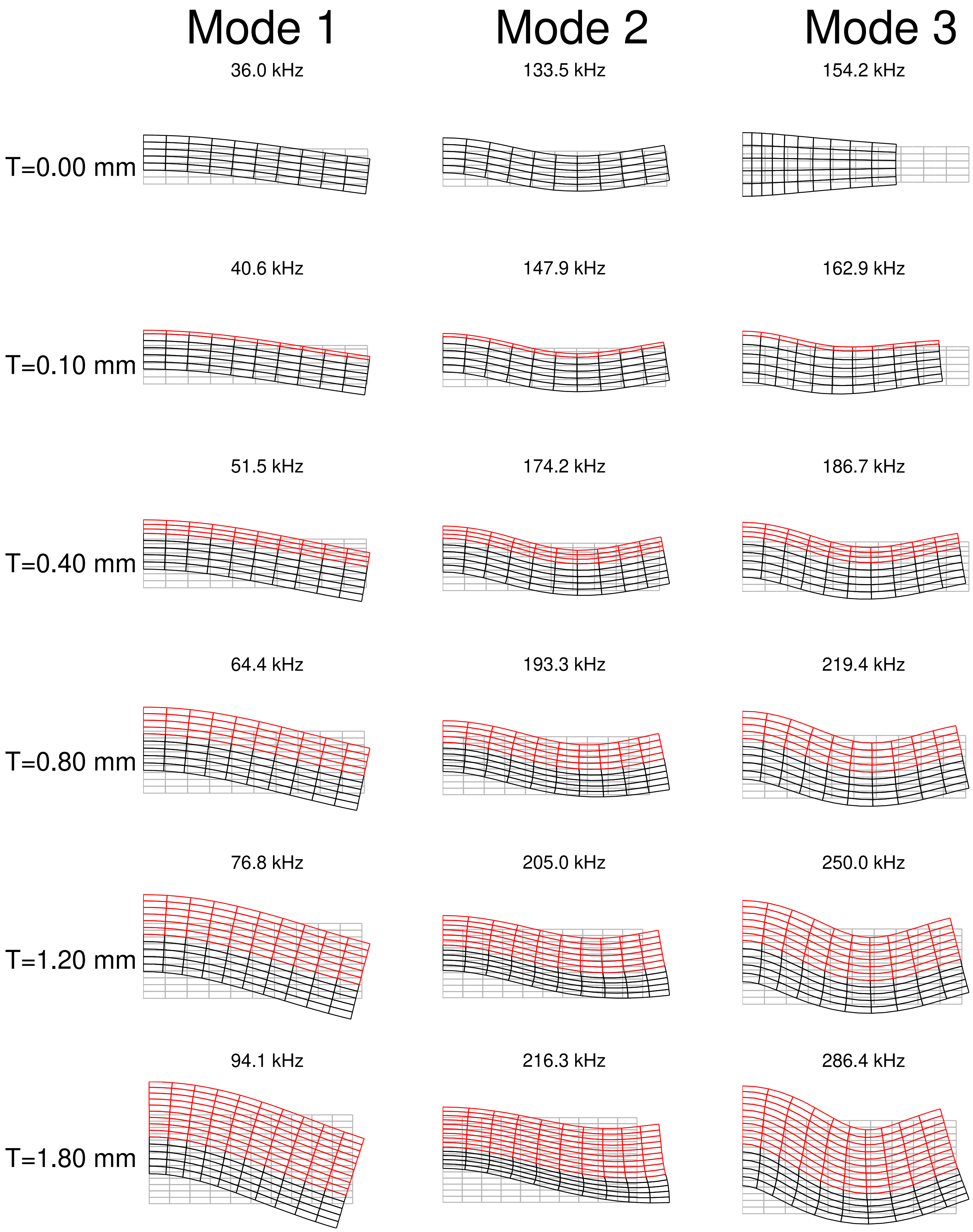
- Number of elements used in simulations chosen after convergence tests:
  - 5-30 in thickness direction
  - 10 in radial direction

ABAQUS-comparison

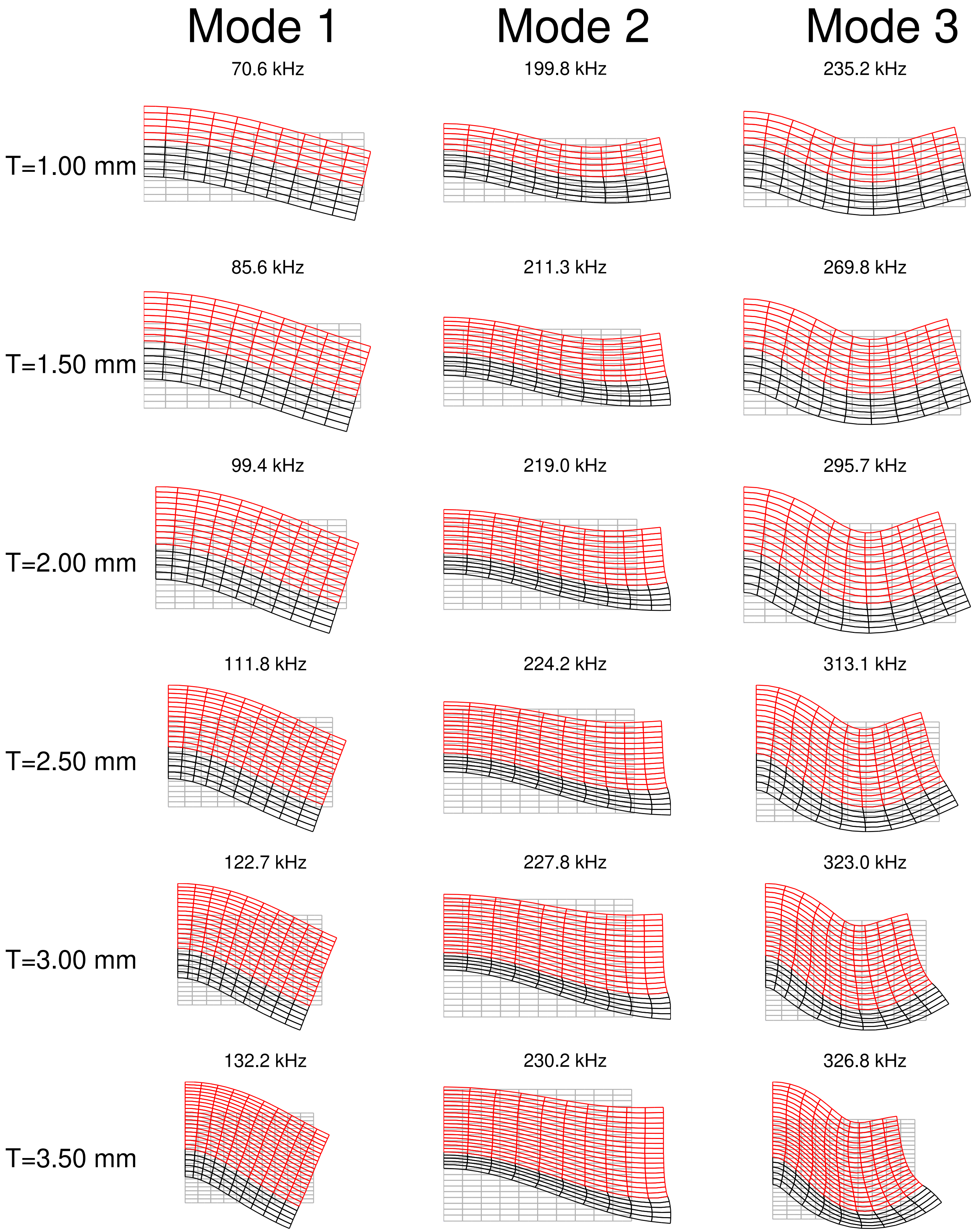
- ABAQUS: commercial FEM-program
- Simulations for PZT5A-disk with titanium layer
  - 15 different simulations, 25 lowest resonance frequencies
  - layer thickness 0.3mm - 12.5mm
- Identical simulation parameters
- Maximum difference: 5ppm
- Example results, $T_e + T_p = 1.337mm$:

FEMP	ABAQUS	DIFF. (PPM)
49.242 kHz	49.242 kHz	-
169.501 kHz	169.501 kHz	-
181.305 kHz	181.305 kHz	-
325.898 kHz	325.898 kHz	-
440.687 kHz	440.687 kHz	-
492.249 kHz	492.249 kHz	-
644.100 kHz	644.100 kHz	-
657.801 kHz	657.801 kHz	-
755.311 kHz	755.311 kHz	-
807.963 kHz	807.963 kHz	-
838.604 kHz	838.604 kHz	-
911.382 kHz	911.382 kHz	-
934.977 kHz	934.977 kHz	-
1011.127 kHz	1011.130 kHz	3.1 ppm
1046.337 kHz	1046.340 kHz	2.8 ppm
1082.437 kHz	1082.440 kHz	2.4 ppm
1101.429 kHz	1101.430 kHz	0.5 ppm
1188.146 kHz	1188.150 kHz	3.2 ppm
1207.442 kHz	1207.440 kHz	1.7 ppm
1252.347 kHz	1252.350 kHz	2.5 ppm
1339.266 kHz	1339.270 kHz	3.1 ppm
1347.758 kHz	1347.760 kHz	1.3 ppm
1379.513 kHz	1379.510 kHz	2.3 ppm
1427.074 kHz	1427.070 kHz	2.9 ppm
1432.201 kHz	1432.200 kHz	0.6 ppm

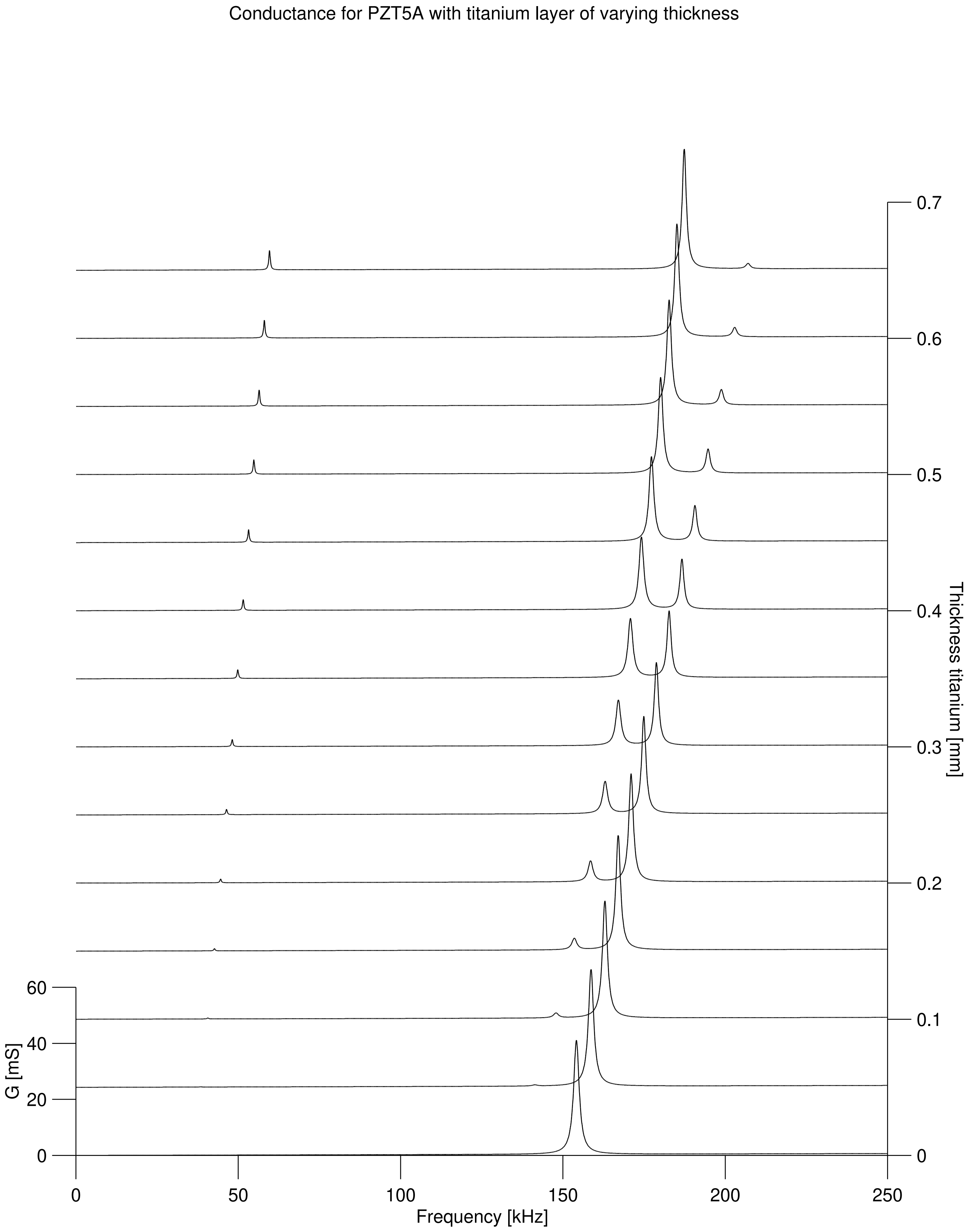
Mode-simulations: PZT5A + Titanium layer



Mode-simulations: PZT5A + Titanium layer

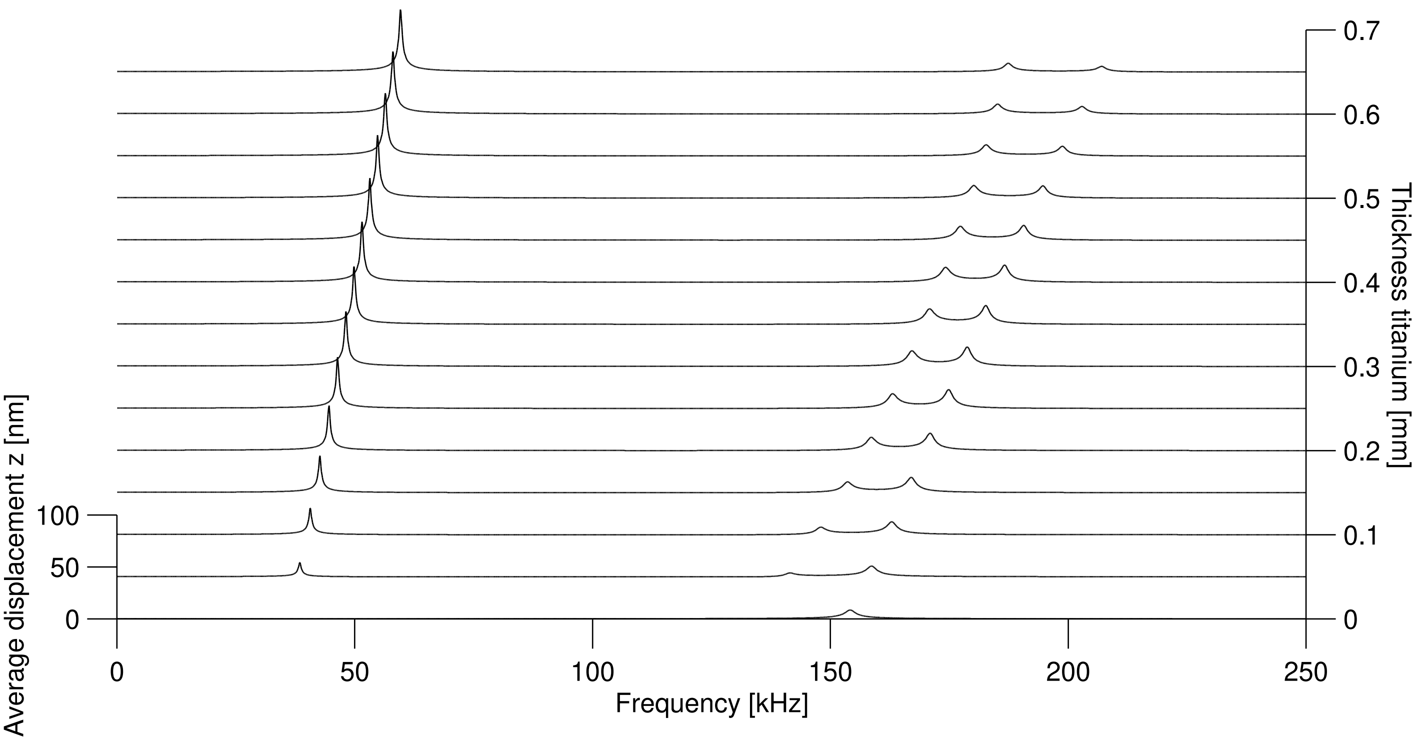


Conductance-simulations: PZT5A + Titanium layer

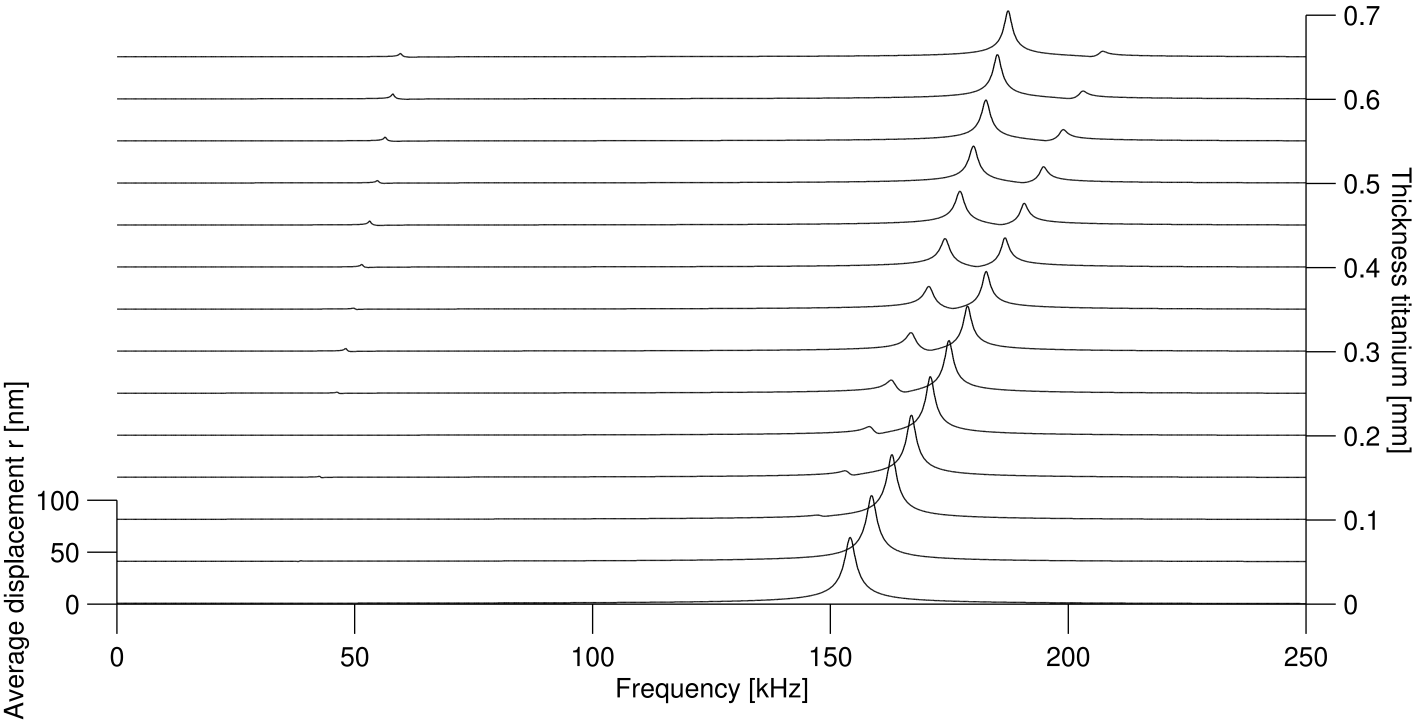


Displacement-simulations: PZT5A + Titanium layer

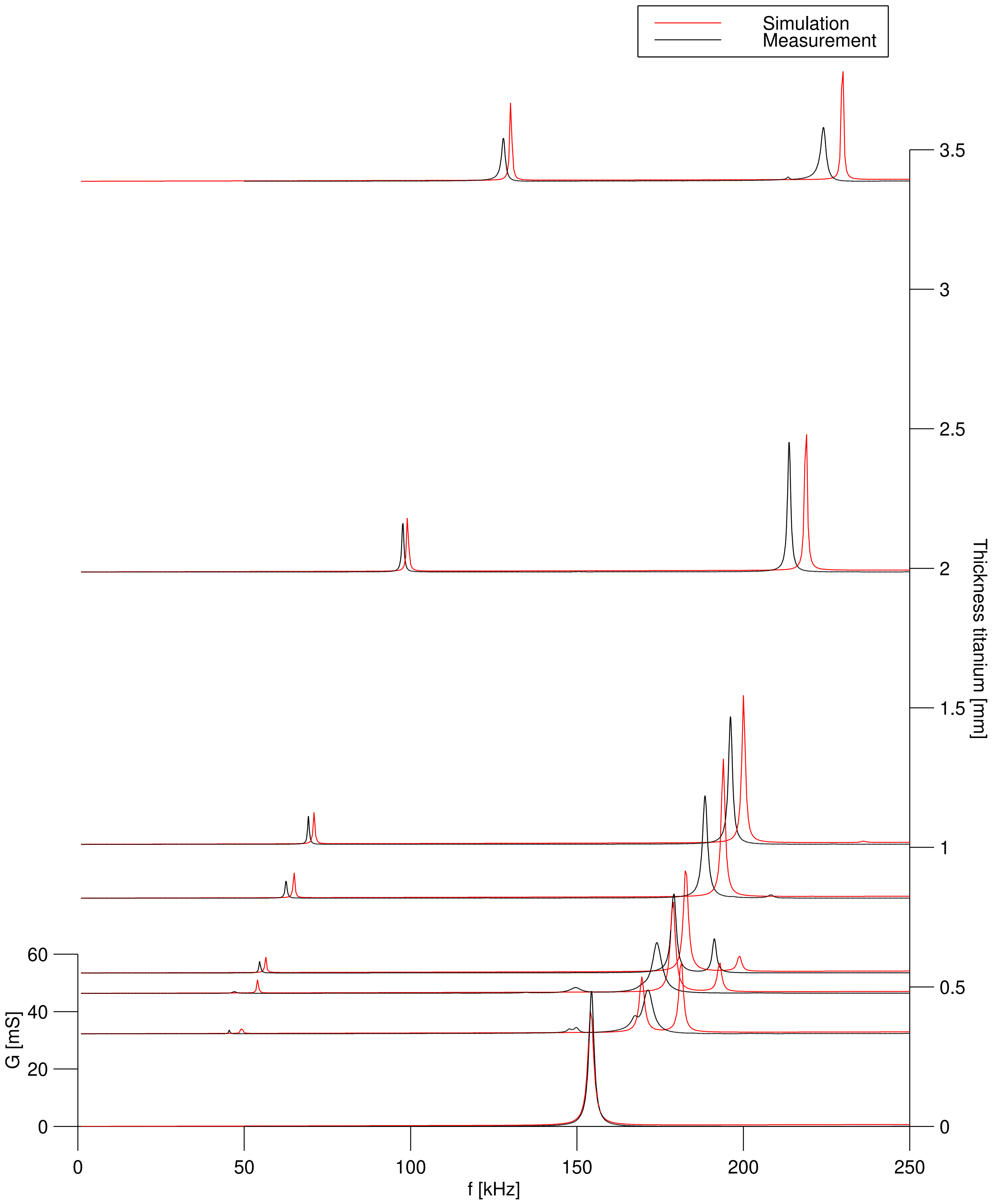
Average displacement, z−direction, PZT5A + Titanium



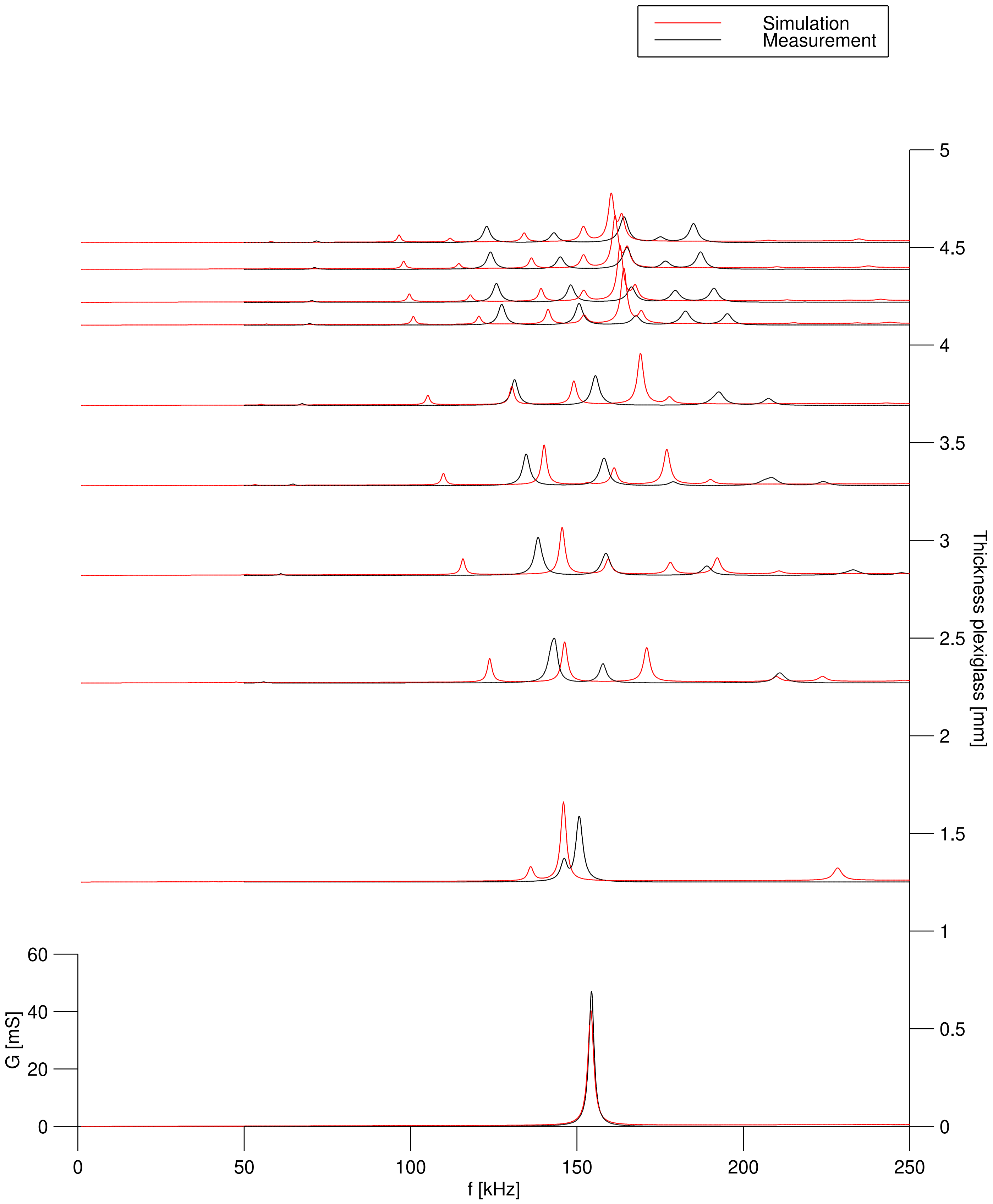
Average displacement, r−direction, PZT5A + Titanium



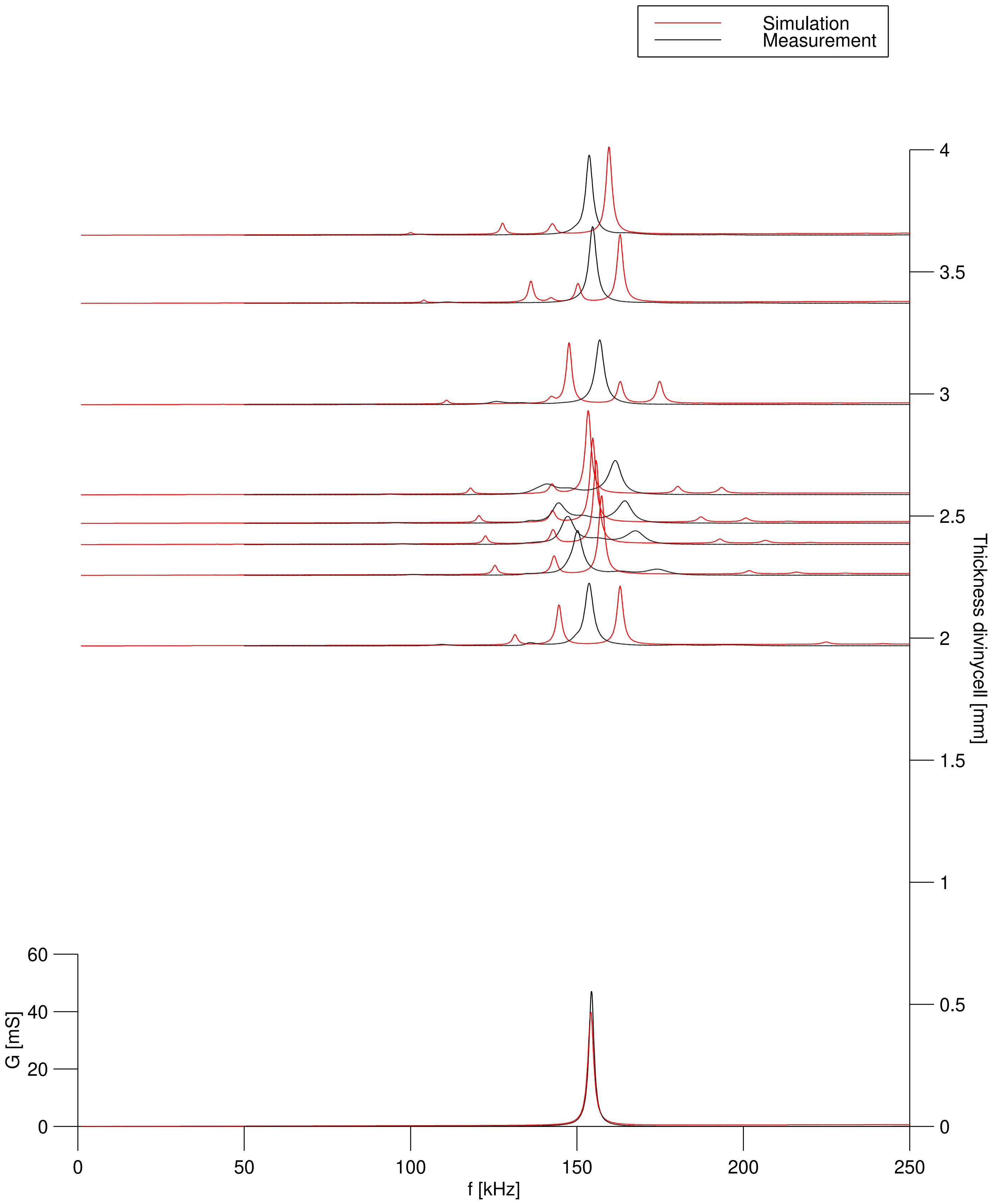
Comparison FEM/measurement, (Titanium)



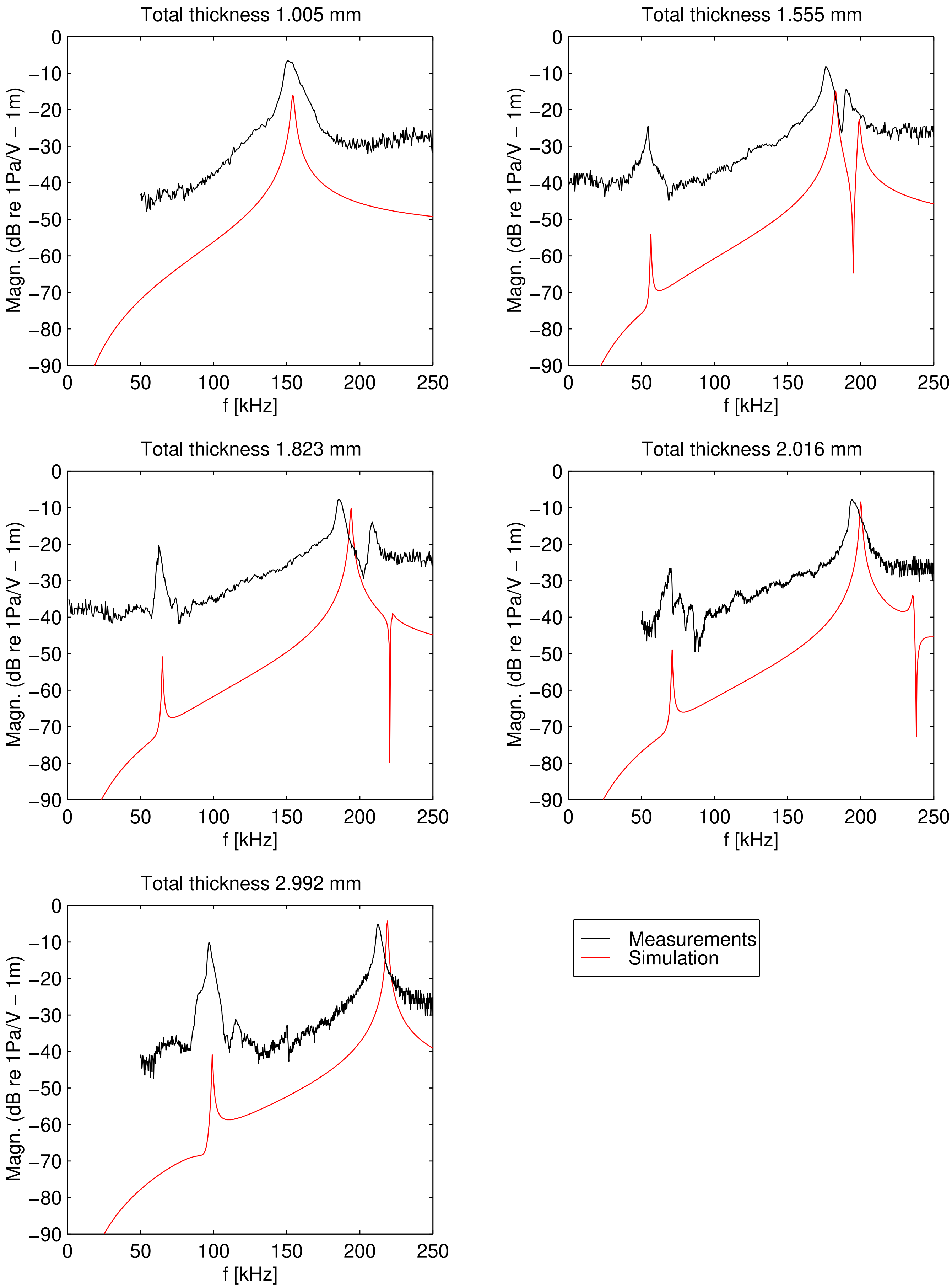
Comparison FEM/measurement, (Plexiglass)



Comparison FEM/measurement, (Divinycell)



Voltage Source Sensitivity: PZT5A + Titanium layer



Conclusions

- Status report on work in progress
- Developed FEM program for piezoceramic disk + front layer:
  - Very good quantitative agreement with ABAQUS
  - Good qualitative agreement with electrical measurements for Titanium-case
  - More work required for plexiglass/divinycell and radiated soundfield